Spin Singlet Quantum Hall Effect and Nonabelian Landau-Ginzburg Theory

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ABSTRACT:

In this paper we present a theory of Singlet Quantum Hall Effect (SQHE). We show that the Halperin-Haldane SQHE wave function can be written in the form of a product of a wave function for charged semions in a magnetic field and a wave function for the Chiral Spin Liquid of neutral spin-$\frac{1}{2}$ semions. We introduce field-theoretic model in which the electron operators are factorized in terms of charged spinless semions (holons) and neutral spin-$\frac{1}{2}$ semions (spinons). Broken time reversal symmetry and short ranged spin correlations lead to $SU(2)_{k=1}$ Chern-Simons term in Landau-Ginzburg action for SQHE phase. We construct appropriate coherent states for SQHE phase and show the existence of $SU(2)$ valued gauge potential. This potential appears as a result of "spin rigidity" of the ground state against any displacements of nodes of wave function from positions of the particles and reflects the nontrivial monodromy in the presence of these displacements. We argue that topological structure of $SU(2)_{k=1}$ Chern-Simons theory unambiguously dictates *semion* statistics of spinons.

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CONTENT

I.INTRODUCTION.

i) General remarks

It has been assumed from the beginning of the theory of Fractional Quantum Hall Effect (FQHE), that the magnetic field, which has to be strong enough to produce the relevant Landau quantization, leads to large Zeeman splitting. Large body of physical theories of FQHE assumed spins of electrons to be polarized completely (which is equivalent to consideration of spinless electrons in the lowest Landau level).

It has been pointed out first by Halperin[1] that this is not always the case. Zeeman splitting is given by $\mathrm{E}_{Zeeman} = g \cdot \mu_B \cdot H$, and Larmour energy is $\mathrm{E}_{Larmour} = eH/\hbar c$. The ratio of these two energies depends on the factor $\mathrm{E}_{Zeeman}/E_{Larmour} = g \cdot \frac{m^*}{m_o}$, where m^* is the effective mass of electron, and g - is the g-factor. The ratio of m^*/m_o in the $\mathrm{Si/SiO_2}$ structures is quite small $m^*/m_o \simeq 0.07$, and g can be as low as 1/4.

We find, thus, that at least in low enough magnetic fields B $\sim$ 1 T, for some materials the ratio $\frac{E_{Zeeman}}{E_{Larmour}} \simeq 0.017$ is quite small. Thus it is a good approximation in this case to neglect Zeeman splitting and consider all states in the Hilbert space of the problem as doubly degenerate due to spin.

Within these assumptions one has to consider the spin unpolarized QHE phase. We will consider below the case of spin singlet QHE phase (SQHE).

Experimentally there is evidence that spin singlet QHE phases are present at some filling factors, see for example.[2]

In this article we will consider the Landau-Ginzburg theory of singlet QHE and how it is connected with nonabelian, namely $\mathrm{SU}(2)_{k=1}$ for spin S=1/2, Chern Simons theory as a natural generalization of the Chern Simons theory for spin polarized case. We will show how the SU(2) valued gauge potential naturally appears in the context of spin coherent states for SQHE [3].

But before considering spin unpolarized case we will summarize briefly the most important features of the Landau-Ginzburg theory for spin polarized case.

II. SPIN SINGLET QUANTUM HALL EFFECT AND LANDAU-GINZBURG THEORY.

i) Halperin-Haldane Wave Function of SQHE and Slave Semion Decomposition.

In this paragraph we consider the physical properties of the singlet Quantum Hall Effect states, given by the Halperin-Haldane wave function [1],[9]:

$$\Psi_m([z_i^+],[z_i^-]) = \prod_{i<j}(z_i^+ - z_j^+)^{m+1}(z_i^- - z_j^-)^{m+1}(z_i^+ - z_j^-)^m e^{-\frac{1}{4}\sum_i |z_i^+|^2 - \frac{1}{4}\sum_i |z_i^-|^2}. \qquad (II.1)$$

where the set of coordinates $z_i^+, i = 1,\ldots,N$ corresponds to the spin $\uparrow$ electrons, and $z_i^-, i = 1,\ldots,N$ corresponds to the spin $\downarrow$ electrons and m is an even integer. In this case $\Psi_m([z_i^+],[z_i^-])$ satisfies the Fock cyclicity condition. In this state, the eigenvalue of the total spin operator is $S = 0$ and the z-component of the spin also has eigenvalue $S_z = 0$. This kind of wave functions naturally appears in the consideration of the spin unpolarized states in the Quantum Hall Effect (QHE) phase.

In contrast to the spin polarized states, in this case we need to describe the charge sector of the SQHE phase as well as the spin sector. By inspecting the structure of this wave function one finds that it has the simple but very important property that the spin and charge degrees of freedom are factorized. The total wave function $\Psi_m([z_i^+],[z_i^-])$ can be written as a product of the charge wave function $\Psi_m^{(1)}([z_i^+],[z_i^-])$ and spin wave function $\Psi^{(2)}([z_i^+],[z_i^-])$. Below we will discuss the properties of the charge and spin wave functions separately. At the end we will put them together again by imposing the constraint that the positions of the charges coincides with those of the spins. This property is strongly reminiscent of the charge and spin separation present in models of Strongly Correlated Electron systems in the context of theories of high temperature superconductors[10].

The wave function is factorized in the following manner [3]:

$$\Psi_m([z_i^+],[z_i^-]) = \Psi^{(2)}([z_i^+],[z_i^-])\Psi_m^{(1)}([z_i^+],[z_i^-]). \qquad (II.2)$$

with

$$\Psi_m^{(1)}([z_i^+],[z_i^-]) = \prod_{i<j}(z_i^+ - z_j^+)^{m+1/2}(z_i^- - z_j^-)^{m+1/2}(z_i^+ - z_i^-)^{m+1/2} e^{-\frac{1}{4}\sum_i |z_i^+|^2 - \frac{1}{4}\sum_i |z_i^-|^2}. \qquad (II.3)$$

symmetry. Fortunately such an approach does exist: it is the non-abelian SU(2) CS theory. A non-abelian CS term, much like the abelian CS theory used in the description of the spin polarized QHE [5,6,7,16], attaches fluxes to particles. But, unlike the "abelian" approach mentioned above, the non-abelian CS theory is invariant under SU(2) rotations of the spin. Furthermore, this invariance is local and the theory is a gauge theory. It turns out that the CS theory represents the only possible local way to attach particles to SU(2) fluxes. Below we will follow this second way in considering the spin wave function.

Consider the set of coordinates $[z_i^+], [z_i^-]$ of a set of some spinors with the spin up components, located at points$[z_i^+]$, and spin down at points $[z_i^-]$. The points $[z_i^+], [z_i^-]$ will be regarded as the positions of sources of an SU(2) field $\wedge_\mu$, taken in the fundamental representation. It corresponds to the spin 1/2 of the electrons , constituing the QHE state. The Lagrangian $\mathcal{L}_{\rm spin}$ of the spin sector is given by Eq.(II.28) with the full non-abelian Chern-Simons term.

The points at which the excitations are located are the the sources for the gauge field . As it can be seen from the variation of the Lagrangian (II.28) over A_0^a:

$$\frac{\delta \mathcal{L}_{\rm spin}}{\delta A_0^a} = \wedge^+ \sigma^a \wedge + \frac{k}{\pi} F_{xy}^a = 0. \qquad (II.30)$$

The strength of the gauge field is given by $F_{xy}^a = \partial_x A_y^a - \partial_y A_x^a + [A_x, A_y]^a$. Let us assume that the particles have a mass m. The path-integral representation of a matrix element of the evolution operator is given as a sum over all possible particle trajectories and gauge field histories. The constraint of Eq.(II.30) requires that each term in this amplitude should contain a factor representing a path-ordered exponential of the $SU(2)$ gauge field along each particle trajectory. These path-ordered exponentials are usually referred to as Wilson lines. In first quantization, the time evolution during the time interval t of the heavy sources will be given by the amplitude:

$$\Psi([z_i'^+],[z_i'^-],t) = \sum_{Paths} e^{-i\int dt(\sum_i m/2|dz_i^+/dt|^2+\sum_i m/2|dz_i^-/dt|^2)} \int D[A] \otimes_{i,j} W_i(z_i'^+, z_i^+) W_j(z_j'^-, z_j^-) e^{ik\int d^2x dt \mathcal{L}_{CS}} \Psi([z_i^+],[z_i^-$$

$$(II.31)$$

where z'^+, z'^- are the set of final positions of the sources, and

$$W_i(z_i', z_i) = [Pe^{i\int_{z_i}^{z_i'} A_l dx^l}]. \qquad (II.32)$$